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A PARAMETRIC LTR-SOLUTION FOR DISCRETE-TIME SYSTEMS.

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ABSTRACT.

A parametric LTR-solution for discrete-time compensators incorporating filtering observers which achive exact recovery will be presented for both minimum and non-minimum-phase systems.

First the recovery error, which define the difference between the target loop transfer and the full loop transfer function, is manipulated into a general form consisting of the target loop transfer matrix and the fundamental recovery matrix. Based on the recovery matrix a parametric LTR-solution will be developed. At last it will be shown that the LQG/LTR solution is included in this new parametric solution as a special case.

1 INTRODUCTION.

The LQG/LTR feedback design methodology for robust model-based compensation for both continuous-time and discrete-time systems has received much attention in the recent years, see f.ex. [1-10].

For continuous-time minimum-phase systems the asymptotic recovery techniques works very effective. Unfortunately a similar asymptotic recovery technique is not feasible in discrete-time. When the LQG/LTR-solution is applied to the discrete-time case, the recovery error will in general remains finite when a prediction observer is used, [6]. However, it is still possible to make LTR, but other methods must be applied. Based on the equation for the recovery error, a parametric exact LTR-solution has been developed in [9] for minimum-phase as well as non-minimum phase systems.

If the processing time of computing the control signal is negligible in comparison to the sampling interval, a filtering observer can be used in the compensator instead of a predition observer. It is then possible to achieve exact recovery in this case by using the LQG/LTR-solution when the system is minimum phase and has maximal number of zeros [7]. However, when these conditions isn't satisfied the LQG/LTR-solution will result in a finite recovery error, but it will still be possible to make exact recovery by applying the principple from the parametric LTR result in [9] to this case. It will then be possible to make exact recovery for both non-minimum-phase systems as well as for systems with less than the maximal number of zeros.

2 RECOVERY EQUATION.

In the following square discrete-time systems $S(A,B,C)$ are considered. The plant transfer matrix $G(z)$ is given by:

$$G(z) = C\Phi(z)B \quad , \dim G(z) = m \times m \qquad (1)$$
$$\Phi(z) = (Iz-A)^{-1}, \dim \Phi(z) = n \times n$$

It will be assumed that the model is minimal and let the number of transmission zeros be p. The plant is controlled by using a feedback compensator consisting of a filtering observer in serie with a state-feedback controller.

The compensator transfer matrix $H(z)$ can be rewritten as [7]:

$$H(z) = zK(Iz - (I - F^f C)(A - BK))^{-1}F^f \qquad (2)$$

where F is the full-order observer gain, F^f is the filtering observer gain (the relation between F and F^f is: $F = AF^f$, [7]) and K is the full-state feedback gain.

Now let the two step LTR-design method [1,2,3,6,7] be used for the design of $H(z)$ which consist of a target design of one of the compensator gains (K or F) followed by a recovery design of the other gain.

In order to formulate the loop-shape robustness constraints the uncertainties (disturbance, noise and modelling errors) are reflected to the plant input [6]. The target loop transfer function is then the full-state loop transfer $K\Phi B$ and the full loop transfer is HG [2,3]. Let $E_I(z)$ define the recovery error as [6,9]:

$$E_I(z) = K\Phi(z)B - H(z)G(z) \qquad (3)$$

In order to have exact recovery it is required that $E_I(z) = 0$ for all z. Now let eq. (3) be rewritten in an equvialent form:

$$E_I(z) = M_I(z)[I + M_I(z)]^{-1}[I + K\Phi(z)B] \qquad (4)$$

where $M_I(z)$ is the recovery matrix defined by:

$$M_I(z) = K^f(A - FC)(Iz - A + FC)^{-1}B \qquad (5)$$

where $K = K^f A$.

Proof of eq. (4) is given in the appendix.

Eq. (4) for the recovery error is equvialent to the equation defined in [6,9] when a prediction observer is used instead. The only difference is the equation for the recovery matrix.

It is simple to see by using the form of $E_I(z)$ in eq. (4) that:

$$E_I(z) = 0 \text{ iff } M_I(z) = 0 \qquad (6)$$

The recovery matrix M_I will only be used for making exact recovery in this paper, but M_I have also other attractive properties. By using eq. (3) and (4) it is easy to derive that [10]:

$$S_I(z) = S_{TFL}(z)(I + M_I(z)) \qquad (7)$$

where S_I and S_{TFL} are the input sensitivity functions for the full loop and the target full loop transfer function. For more details, please see [10].

3 THE PARAMETRIC LTR-SOLUTION.

Now let the equation for the recovery matrix, eq. (5), be rewritten in the residual form, [6]:

$$M_I(z) = K^f V W^T(Iz - V\Lambda W^T)^{-1}B$$
$$= K^f \sum_{i=1}^{n} \frac{v_i \tau_i w_i^T B}{z - \tau_i} \qquad (8)$$

where $V = [v_1,\dots,v_n]$, $W = [w_1,\dots,w_n]$, $\Lambda = \mathrm{diag}(\tau_1,\dots,\tau_n)$, v_i and w_i^T are right and left eigenvectors associated with the eigenvalue τ_i of $A-FC$. $VW^T = W^TV = I$

It is then easy to show that:

$$M_I(z) = 0 \text{ iff}$$
$$K^f v_i = 0 \text{ or } \tau_i = 0 \text{ or } w_i^T B = 0,\ i = 1,\dots,n \qquad (9)$$

if $A - FC$ is non-defective.

From eigenstructure assignment it is known that the left eigenvector w_i^T with the eigenvalue τ_i of $A-FC$ are given by [11]:

$$[w_i^T \ \ z_i^T]\begin{bmatrix}\tau_i I - A \\ -C\end{bmatrix} = 0,\ i = 1,\dots,n \qquad (10)$$

$$w_i^T F = -z_i^T$$

The last condition in eq. (9) implies that:

$$[w_{i0}^T \quad z_{i0}^T] \begin{bmatrix} \tau_{i0}I - A & B \\ -C & 0 \end{bmatrix} = 0 \tag{11}$$

Maximally p eigenvectors w_{i0}^T can satisfy this condition, if τ_{i0} is selected as a transmission zero of S(A,B,C), [14]. Let these p eigenvalues/vectors be selected from eq. (11) so the p conditions in eq. (9) are satisfied.

The second condition in eq. (9), $\tau_i = 0$, can be satisfied by placing maximally m eigenvalues τ_i at the origin.

Again from eigenstructure assignment [11] it is easily found that with $\tau_i = 0$:

$$w_i^T = z_i^T CA^{-1}, \quad i = p+1,\dots,p+m \tag{12}$$

The choice of z_i^T depends on the type of the system which is treaded: A square uniform rank system or a square non-uniform rank system.

The non-uniform rank case.

The square non-uniform rank case is defined as:

$$CA^iB = 0,\ i = 1,\dots,\alpha-2 \quad CA^{\alpha-1}B \neq 0,\ \det[CA^{\alpha-1}B] = 0 \tag{13}$$

The m eigenvectores w_{i0}^T corresponding to the m zero eigenvalues are given by, [13]:

$$w_{i0}^T = e_{i-p}^T ECA^{-1}, \quad i = p+1,\dots,p+m \tag{14}$$

where E is a m x m non-singular matrix of the appropriate coefficients of the linear combination of the rows of CA^{-1}, see Shaked [13] for calculating E.

Again F can be parametrized by:

$$F = \begin{bmatrix} w_1^T \\ \vdots \\ w_n^T \end{bmatrix}^{-1} \begin{bmatrix} z_1^T \\ \vdots \\ z_n^T \end{bmatrix} \tag{15}$$

$$z_i^T = z_{i0}^T, \quad w_i^T = z_{i0}^T C(\tau_{i0}I - A)^{-1}, \quad i = 1,\dots,p$$

$$z_i^T = e_{i-p}^T E, \quad w_i^T = z_i^T CA^{-1}, \qquad i = p+1,\dots,p+m$$

and $(\tau_i, z_i^T,\ i=p+m+1,\dots,n)$ are free design parameters.

The uniform rank system.

The square uniform rank case is defined as, see [13]:

$$CA^iB = 0,\ i = 1,\dots,\alpha-2,\ \det[CA^{\alpha-1}B] \neq 0 \tag{16}$$

The solution for the uniform rank case is given by eq. (15) with E = I.

If p < n-m the remaining n-p-m conditions in eq. (9) must be satisfied by selecting K^f suitable. The first condition in eq. (9) implies:

$$K^f[v_1,\dots,v_n] = [\Omega\ 0] \tag{17}$$

with dim Ω = m x (p+m) but otherwise it is arbitrary. Now

$$K = [\Omega\ 0]V^{-1}A = \Omega \begin{bmatrix} w_{10}^T \\ \vdots \\ w_{p+m0}^T \end{bmatrix} A = \Omega\Gamma A \tag{18}$$

with dim Γ = (p+m) x m. Γ consist of the left eigenvectors w_{i0}^T constrained in eq. (11) and in eq. (14) and is thus a matrix of fixed elements.

A simple parametrization of the controller matrices which achieves exact recovery are found. An equivalent LTR result can be found in [9] when a prediction or a minimal-order observer are used.

4 DISCUSSION.

In this section a few important consequences of exact LTR are discussed.

If the rank[CB] = m, i.e. S(A,B,C) have maximal number of zeros, and S(A,B,C) is minimum-phase, the n recovery conditions in eq. (9) can be satisfy by the selection of F, and K will be free to select. A more simple equation for F than eq. (15) is given by:

$$F = AB(CB)^{-1} \tag{19}$$

which is also the LQG/LTR solution [7] , which show that the LQG/LTR solution is included as a special case.

If the LTR-results from sec. 3 are used on a non-minimum phase system G(z), the resulting controller will be unstable. It is, however, still possible to achieve exact recovery for non-minimum phase systems. In order to facilitate exact recovery, note that in the selecting of F, only a subset j of the eigenvectors constrained by eq. (11) need to be chosen. The consequence is, of cause, a reduction of the free parameters in K. Therefore such a selection is only advisible for non-minimum phase system if only the system's j minimum phase zeros are used in eq. (11). By doing this the following equation will be satisfied by a stable compensator:

$$K\Phi(z)B = H(z)G(z) \tag{20}$$

The non-minimum phase zeros of G(z) are not cancelled out on the right hand side. Hence HG and KΦB are both non-minimum phase. This result is in agreement with the result in [15].

The selecting af F is only constrained by eq. (15) and stability can always be achived for the observer. The parametrization of the state-feedback matrix K imply that all the closed loop eigenvalues cannot be assigned freely, and no stability guarantees are available.

If a prediction observer is used, it is simple to develop dual result for the plant output loop breaking point [6,9], which be due to the duality between the prediction observer and the state-feedback scheme. However, in spite of the missing duality when a filtering observer is used, it is still possible to develop dual result for the plant output loop breaking point. This can easily be shown by applying the observer gain F as the target design instead of K. The recovery error defined at the plant output loop breaking point is then given by:

$$E_O(z) = C\Phi(z))F - G(z)H(z) \tag{21}$$

Eq. (21) can also be rewritten into a form equvialent to eq. (4).

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APPENDIX.

Proof of eq. (4).

$$\begin{aligned}
E_I(z) &= K[I-z(Iz-(I-F^fC)(A-BK))^{-1}F^fC](Iz-A)^{-1}B \\
&= K(Iz-(I-F^fC)(A-BK))^{-1}[Iz-(I-F^fC)(A-BK)-zF^fC](Iz-A)^{-1}B \\
&= K(Iz-(I-F^fC)(A-BK))^{-1}(I-F^fC)(Iz-A+BK)(Iz-A)^{-1}B \\
&= K^f(Iz-(A-FC)(I-BK^f))^{-1}(A-FC)(B+BK(Iz-A)^{-1}B) \\
&= K^f(A-FC)(Iz-(I-BK^f)(A-FC))^{-1}B(I+K\Phi(z)B) \\
&= K^f(A-FC)(Iz-A+FC)^{-1}(I+BK^f(A-FC)(Iz-A+FC)^{-1})^{-1}B(I+K\Phi B) \\
&= K^f(A-FC)(Iz-A+FC)^{-1}B(I+K^f(A-FC)(Iz-A+FC)^{-1}B)^{-1}(I+K\Phi B) \\
&= M_I(z)(I+M_I(z))^{-1}(I+K\Phi B)
\end{aligned}$$